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Mode self-locking in gas lasers

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Abstract. The Lamb theory of mode self-locking in gas lasers is approximated to provide criteria for self-locking to occur in practical systems. It is predicted that self-locking should be fairly easily attained in the He–Ne 1·15 μm and 0·633 μm lasers, and this is confirmed by experiment.

1. Introduction

The phenomenon of mode self-locking in a gas laser was apparently first observed by Javan (private communication to Lamb 1964). Self-locking in a He–Ne 0·633 μm laser was later observed by Crowell (1965) and similar effects in an Ar^+ laser operating at 0·488 μm have been reported by Gaddy and Schaefer (1966).

In semiclassical terms, a gas laser can be considered as a cavity oscillator producing oscillation in a number of transverse electromagnetic standing wave modes, each of which can be described by the product of a space-varying term and a time-varying term. At any point in the cavity the amplitude of the electric vector of the field can be defined by

$$E(\boldsymbol{r}) = \sum_{nmq} A_{nmq}(t) U_{nmq}(\boldsymbol{r}).$$

The indices n, m and q define a particular mode of oscillation. In general, the $U(\boldsymbol{r})$ terms depend only upon the dimensions of the cavity. For a passive cavity, the time-dependent components $A_{nmq}(t)$ will have no fixed phase relationship from one mode to the next. However, if oscillation in the cavity is maintained by an induced electric polarization term, the situation may occur when a time-independent phase relationship is set up between the modes. The modes are then said to be locked. As far as the output of the laser is concerned, locking has the important effect that constructive interference can take place between the various frequency components of the laser output. In some cases an output consisting of a train of pulses may result.

Locking is most commonly produced by the insertion of a time-varying loss into the cavity (see Crowell 1965), but may also occur spontaneously. Statz (1967) has calculated the conditions for such self-locking to occur and suggests that in gas lasers it should be a fairly rare occurrence. In the present work, calculations employing the semiclassical theory of Lamb (1964) are used to derive approximate expressions for the conditions under which self-locking can occur in a gas laser. The results suggest that self-locking should be a fairly common occurrence, and this agrees with experiments performed on He–Ne 0·633 μm and 1·15 μm lasers.

2. Theory

The type of mode self-locking described above essentially describes three or more infinitely long wave trains adding together in constructive interference. Simple wave theory shows that this can only occur when the wave frequencies form a simple arithmetical progression and the phase terms have a time-independent relationship. In a passive laser cavity the mode frequencies can be defined to a good approximation by the equation

$$\Omega_q = \frac{qc}{2d} = q\Delta$$

where c is the velocity of light, d the length of cavity, q an integer and Δ the frequency interval between modes. The frequency condition for self-locking holds for this case, but there are no restrictions on the relative phase and so locking does not occur. When the cavity contains an active medium, the frequencies of oscillation are 'pulled' from the passive cavity resonance owing to the frequency dependence of the gain. In general, neither locking condition will hold. However, for a medium which is inhomogeneously broadened, Lamb showed that the conditions for locking can occur by virtue of the fact that 'combination tones' are induced in the medium at frequencies close to the passive cavity frequencies. Under certain conditions there is a strong interaction between these 'tones' and the pulled cavity resonance, such that oscillation occurs preferentially at the combination tone frequency. In this situation the frequency condition for locking is satisfied. The phase condition will also be satisfied and so self-locking can occur.

Lamb derived equations for the field strength, frequencies of oscillation and relative phases of three modes oscillating simultaneously in a laser. The main equations of interest in the present case are those determining the frequencies ν_1, ν_2, ν_3 and phases ϕ_1, ϕ_2, ϕ_3 of three modes of amplitude E_1, E_2 and E_3. These are given by equations of the form

$$\nu_1+\dot{\phi}_1 = \Omega_1+\sigma_1+\rho_1 E_1^2+\tau_{12}E_2^2+\tau_{13}E_3^2-E_2^2E_3E_1^{-1}(\eta_{23}\sin\psi-\epsilon_{23}\cos\psi) \tag{1}$$

$$\nu_2+\dot{\phi}_2 = \Omega_2+\sigma_2+\rho_2 E_2^2+\tau_{21}E_1^2+\tau_{23}E_3^2+E_1E_3(\eta_{13}\sin\psi+\epsilon_{13}\cos\psi) \tag{2}$$

$$\nu_3+\dot{\phi}_3 = \Omega_3+\sigma_3+\rho_3 E_3^2+\tau_{31}E_1^2+\tau_{32}E_2^2-E_2^2E_1E_3^{-1}(\eta_{21}\sin\psi-\epsilon_{21}\cos\psi) \tag{3}$$

where

$$\psi = (2\nu_2-\nu_1-\nu_3)t+(2\phi_2-\phi_1-\phi_3).$$

The Ω_n terms are the passive cavity resonant frequencies and the σ_n terms describe the pulling of the modes owing to the shape of the Doppler-broadened gain curve. The terms in $\rho_n E_n^2$ describe power-dependent 'pushing' of the mode frequencies and terms in $\tau_{nm}E_m^2$ describe 'mode repulsion' effects between modes. The η_{nm} and ϵ_{nm} terms are produced by combination tones in the active medium, and ψ defines the 'relative phase angle' between the three modes. From the definition of locking given above, it can be seen that in the present case self-locking occurs when ψ is time-independent.

By manipulation of equations (1) to (3) a differential equation for ψ can be produced:

$$\dot{\psi} = \sigma+A\sin\psi+B\cos\psi \tag{4}$$

where

$$\sigma = (2\sigma_2-\sigma_1-\sigma_3)+(2\rho_2E_2^2-\rho_1E_1^2-\rho_3E_3^2)+\text{interaction terms}$$

$$A = -(2E_1E_3\eta_{13}+E_2^2E_1E_3^{-1}\eta_{21}+E_2^2E_3E_1^{-1}\eta_{23})$$

$$B = -(2E_1E_3\epsilon_{13}-E_2^2E_3E_1^{-1}\epsilon_{23}-E_2^2E_1E_3^{-1}\epsilon_{21}).$$

Lamb demonstrated that two situations exist: If $A^2+B^2 < \sigma^2$, ψ is a linear function of time, and the modes are not locked. If $A^2+B^2 > \sigma^2$, a pole exists in the integral solution of (4) and ψ cannot have a linear dependence on time. Hence $2\nu_2-\nu_1-\nu_3$ must approach zero rapidly and

$$\nu_3-\nu_2 = \nu_2-\nu_1.$$

The relative phase rapidly approaches a constant value which may or may not be zero, depending on the value of σ. The modes are therefore locked. It should be noted that only if the relative phase angle is close to zero will sharply defined trains of pulses be produced.

Thus the necessary criterion for self-locking is given by

$$(A^2+B^2)^{1/2} \geqslant |\sigma|. \tag{5}$$

The expressions for A, B and σ given by Lamb are of considerable complexity even for the relatively simple case of three-mode operation. In particular, it can be seen that they depend on the values of the field amplitudes which are defined by three further equations of the form (1)–(3). An analytical solution of the problem thus involves the solution of six simultaneous differential equations. Approximate solutions which will be of use in deciding the possible conditions for mode locking may be obtained by making certain simplifying assumptions.

To a first order of approximation, it is possible to take the mode amplitudes as being equal to their 'uncoupled' values (setting all coupling terms equal to zero):

$$E_n = (\alpha_n/\beta_n)^{1/2}$$

where α_n describes the small-signal gain of the system, β_n describes saturation, and expressions for both terms are given by Lamb. Assuming that $\Omega_n - \omega \ll \Delta\nu_{\rm D}$, the Doppler width, and that $N_2 \ll \bar{N}$, a simple expression for $(A^2+B^2)^{1/2}$ may be obtained. For experimental purposes this is best written for the case of three modes in terms of the loss $L_{\rm m}$ which must be inserted into the cavity to extinguish the laser, and the inserted loss L required to produce a given laser intensity:

$$(A^2+B^2)^{1/2} = \frac{\gamma_{\rm a}\gamma_{\rm b}}{\pi\Delta}(L_{\rm m}-L)\left(\frac{2({\gamma_{\rm ab}}^2+\Delta^2)}{2{\gamma_{\rm ab}}^2+\Delta^2} - \frac{{\gamma_{\rm ab}}^2+a^2\Delta^2}{2{\gamma_{\rm ab}}^2+a^2\Delta^2}\right) \quad (6)$$

where $\gamma_{\rm ab} = \frac{1}{2}(\gamma_{\rm a}+\gamma_{\rm b})$, as defined by Lamb. In this case the centre mode is considered to be displaced from the centre of the gain curve by an amount $a\Delta$.

The σ term can be evaluated to the same order of approximation, and is found to fall into two parts. One, due to pulling, is dependent only upon the shape of the gain curve and is given by the σ_n terms in equation (4):

$$2\sigma_2-\sigma_1-\sigma_3 = \frac{6a\Delta^4}{\pi(0{\cdot}6\Delta\nu_{\rm D})^3}L_{\rm m}. \quad (7)$$

The intensity-dependent pushing term is given from equation (4) by

$$\rho = 2\rho_2{E_2}^2-\rho_1{E_1}^2-\rho_3{E_3}^2+\text{interaction terms}.$$

To the present order of approximation

$$\rho = \frac{\gamma_{\rm ab}}{\pi}(L_{\rm m}-L)\left(\frac{a\Delta^4\{\Delta^2(1-a^2)+4{\gamma_{\rm ab}}^2\}}{(2{\gamma_{\rm ab}}^2+a^2\Delta^2)\{2{\gamma_{\rm ab}}^2+(a+1)^2\Delta^2\}\{2{\gamma_{\rm ab}}^2+(a-1)^2\Delta^2\}}-\tau\right). \quad (8)$$

τ is a complicated function of $\gamma_{\rm ab}$, a and Δ, describing repulsion effects between the holes. It is small and slowly varying compared with the pushing term in ρ. Substituting from equations (6), (7) and (8), it is possible to write down an explicit form for the locking criterion defined in (5). For comparison between theory and experiment, it is useful to write this explicitly in terms of the measurable quantities $L_{\rm m}$ and L in the form

$$A'(L_{\rm m}-L) \geqslant \sigma' L_{\rm m}+\rho'(L_{\rm m}-L) \quad (9)$$

where A' is defined from (6) as $(A^2+B^2)^{1/2}/(L_{\rm m}-L)$, σ' is defined from (7) as $(2\sigma_2-\sigma_1-\sigma_3)/L_{\rm m}$ and ρ' is defined from (8) as $\rho/(L_{\rm m}-L)$.

All the above equations have been calculated in the approximation $\gamma_{\rm ab}$, $\Delta \ll \Delta\nu_{\rm D}$, which might be expected to be valid for most gas laser systems. Only three modes are considered and interaction terms are only included to the first order.

If more than three modes are considered, new interaction terms appear and can be calculated from Lamb's theory. The effect of further modes on the self-locking criterion given by (5) for the three central modes can be discussed qualitatively. As far as the right-hand side of equation (5) is concerned, the pulling terms are unaffected. The pushing terms would be increased slightly since the mode repulsion effect would be counterbalanced by repulsion between the middle three modes and the outer ones.

The effect on the left-hand terms is more complicated. New contributions to A and B will occur but these will depend on the amplitudes of the weakest modes and so might be expected to be small. Thus the three-mode approximation might be expected to give tolerably acceptable results for quite a wide range of operation.

The 'non-interacting' form of the field amplitudes is a more important source of error. This approximation will be worst when interactions are large, such as when there are distributions of modes symmetric with respect to the centre of the gain curve. The pulling of the modes will be unaffected, but the interaction terms in A and B and those in the expressions for pushing will increase. An alternative result for any particular set of values of a, γ_{ab} and Δ might be approximated by taking values for the amplitude E_n calculated on the assumption of no locking, and substituting into equation (4). In general, assuming that mode competition does not make three-mode operation impossible, self-locking will tend to become more likely as the interaction terms increase. This is expressed approximately in the simple analysis by the fact that for a symmetrical disposition of modes, when a becomes zero, the right-hand side of equation (9) becomes zero and hence the modes are always self-locked.

Finally, it should be noted that the Lamb theory rests upon third-order perturbation theory and might not be valid for high-gain laser systems.

3. Results of simple theory

Equation (9) describes the way in which the criterion for self-locking is functionally related to the various parameters of the laser system. This criterion can be described schematically as shown in figure 1. If $A' \geqslant \rho' + \sigma'$, there is a value of laser intensity above which self-locking occurs. If $A' < \rho' + \sigma'$ self-locking is never possible.

To the order of approximation employed so far, the criterion can be written

$$\frac{4a\Delta^5}{\gamma_a\gamma_b(0\cdot6\Delta\nu)^3}+\frac{\Delta\gamma_{ab}}{1\cdot5\gamma_a\gamma_b}\left(\frac{\Delta^4 a\{\Delta^2(1-a^2)+4\gamma_{ab}{}^2\}}{(2\gamma_{ab}{}^2+a^2\Delta^2)\{2\gamma_{ab}{}^2+(a+1)^2\Delta^2\}\{2\gamma_{ab}{}^2+(a-1)^2\Delta^2\}}-\tau\right)\leqslant 1. \quad (10)$$

The criterion is highly sensitive to the value of Δ and quite sensitive to a. As mentioned above, when $a = 0$ self-locking will always occur. At other values of a there is a critical value of Δ/γ_{ab} above which no locking is possible.

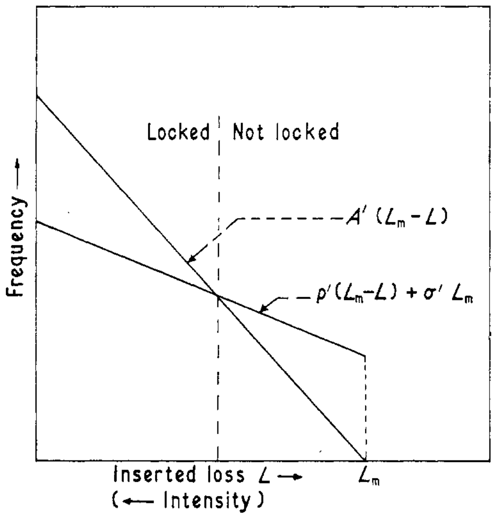

Figure 1. Criterion for self-locking to occur in a gas laser (varying inserted loss). A locked region exists only when $A' > \rho' + \sigma'$.

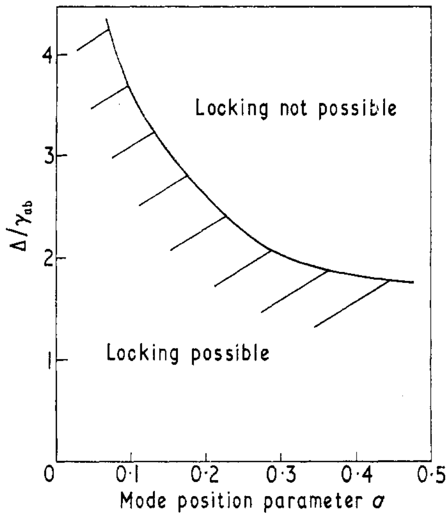

Figure 2. Values of Δ/γ_{ab} for which self-locking is possible plotted as a function of the positions of modes with respect to the centre of the gain curve. Approximate diagram for lasers with $\gamma_{ab} < \Delta$.

To take an example, figure 2 shows the criterion for locking in terms of the ratio Δ/γ_{ab} as a function of a for He–Ne 1·15 μm or 0·633 μm laser lines. It can be seen that in both cases there is a reasonable range of values of a over which locking is possible.

Comparable calculations have been made by Statz using a model not based on Lamb's theory. Statz introduces combination tones via a saturation term differing substantially from Lamb's. His results differ from those of the present work in that the term equivalent to A' is much smaller, suggesting that locking in gas lasers is unlikely except in special cases. This is in disagreement with the results of the Lamb approach.

4. Experiment

There are two possible means of investigating self-locking. Analysis of the laser output in the time domain may be carried out using ultra-fast photomultiplier tubes. When locking occurs, the continuous wave laser output becomes a chain of pulses of sub-nanosecond width. This is the method used by Crowell and by Gaddy and Schaefer. Alternatively, the output may be analysed in the frequency domain by observing beats between modes. This technique, in which the frequencies of the beat notes may be measured as a function of cavity Q, has been described by Allen *et al.* (1969). Self-locking is manifested by the disappearance of all save one of the beat notes at frequencies near $c/2d$ and elimination of all beats at frequencies near $2c/2d$ except one at exactly double the frequency of the remaining lower beat. In He–Ne 1·15 μm and 0·633 μm lasers, self-locking appeared quite spontaneously over a range of axial mode separations of 150–200 MHz. This was presumably due to the thermal drift of the cavity modes from a disposition in which the locking criterion was not satisfied to one in which it was. Locking could also be obtained by carefully tuning the laser mirrors or by decreasing the loss inserted in the cavity. A typical result showing the appearance of self-locking as inserted loss is decreased as shown in figure 3. At a certain value of L, one of the beats near $c/2d$ disappears and the beat

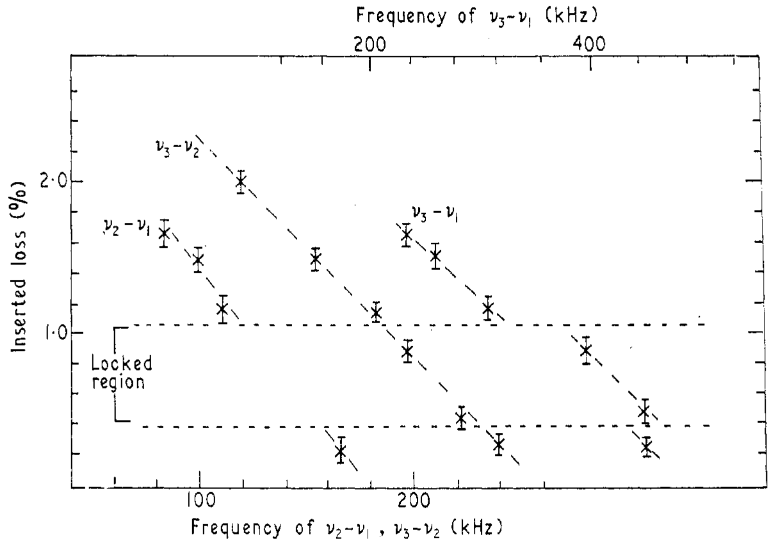

Figure 3. Typical experimental result showing self-locking occurring as inserted loss is decreased in a laser cavity (He–Ne 1·15 μm).

near $2c/2d$ shifts to a frequency exactly double that of the remaining lower frequency beat, to within the resolution of the apparatus (± 500 Hz). As the intensity is further increased the beats become unlocked again, possibly due to thermal drift changing the value of a. The beat frequencies just before locking occurs can be measured. In a typical case of a laser oscillating at 1·15 μm with $L_m = 2{\cdot}6\%$ and $\Delta = 149$ MHz, the observed value of $2\nu_2-\nu_3-\nu_1$ was 67 ± 6 kHz. For a laser oscillating at 0·633 μm with $L_m = 1\%$ and $\Delta = 164$ MHz, the observed value of $2\nu_2-\nu_3-\nu_1$ was 13 ± 1 kHz.

5. Comparison of theory with experiment

The work of Statz suggests that self-locking in a He–Ne laser should be a rare occurrence. In fact, the experiment discussed above suggests that self-locking is common in an unstabilized multimode He–Ne laser. This agrees qualitatively with the results of the Lamb theory, but it is necessary to see if quantitative agreement is obtained.

The criterion for self-locking of equation (5) gives the value for $2\nu_2-\nu_1-\nu_3$ attained just before self-locking occurs:

$$2\nu_2-\nu_3-\nu_1 = \frac{\gamma_a\gamma_b}{\pi\Delta}(L_m-L)\left(\frac{2({\gamma_{ab}}^2+\Delta^2)}{2{\gamma_{ab}}^2+\Delta^2}-\frac{{\gamma_{ab}}^2+a^2\Delta^2}{2{\gamma_{ab}}^2+a^2\Delta^2}\right).$$

The experimental results described earlier can be used to obtain approximate values for the term $\gamma_a\gamma_b$ in this equation (the right-hand side is quite insensitive to the values of γ_{ab} and a in the second bracket).

For $1{\cdot}5$ μm, $\gamma_a\gamma_b = (8\pm1)\times10^{14}$ Hz and for $0{\cdot}633$ μm, $\gamma_a\gamma_b = (6\pm1)\times10^{14}$ Hz.

The value of $\gamma_a\gamma_b$ for the $1{\cdot}15$ μm line agrees well with those measured by other techniques (e.g. Bennett *et al.* 1965), as does that for $0{\cdot}633$ μm (Fork and Pollack 1965). This suggests that the value of A given by the approximation is as accurate as could be expected, and that the combination tone terms given by the Lamb theory are consistent with experimental observation.

Figure 2 demonstrates the dependence of the self-locking criterion on mode distribution and mode spacing. Qualitatively, as the mode spacing increases the possibility of locking becomes less likely, although it is still possible for nearly symmetric tuning (assuming that a symmetric three-mode solution is possible). This was observed experimentally for both $1{\cdot}15$ μm and $0{\cdot}633$ μm. As the cavity length was decreased the time during which the modes were locked also decreased. At the highest value of Δ which could be used (limited by photomultiplier response), the modes remained locked for periods of the order of 10–20 s, or remained unlocked for times of the order of minutes. This agreed with the observation of Allen *et al.* that modes were relatively stable with respect to the gain curve for periods of the order of minutes. With smaller values of Δ, modes could easily be locked and would remain locked for periods of minutes.

The results of the present theory, notably figure 1, appear to suggest the possibility that, given an unlocked three-mode situation, locking can only be obtained by increasing laser intensity. It may be noted that other authors (Crowell 1965, Gaddy and Schaefer 1966) obtained self-locking by a process of decreasing the intensity of their lasers. This can easily be explained by noting that they looked for self-locking by tuning a laser until its output occurred in the form of very sharp pulses. These pulses only occur when the relative phase of the modes is nearly zero, i.e. when $\sigma^2 \ll A^2+B^2$. This will only happen when a is nearly zero, producing a situation where strong competition takes place between the two outer modes. In general, such competition will cause one of the modes to be extinguished, but if the gain of the laser is decreased the intensity is lessened and three-mode operation is possible. Only in this case will the locked modes interact to give a pulse output of the form observed by Crowell.

6. Conclusions

The approximate form of the Lamb theory suggests that mode self-locking is a fairly common event in He–Ne gas lasers. This is in agreement with experimental observations and in disagreement with the results proposed by Statz. The approximate approach gives qualitative insight into the self-locking processes and produces results in agreement with experiment.

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